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A three-dimensional inverse heat conduction problem in estimating surface heat flux by conjugate gradient method

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Abstract

In the present study a three-dimensional (3-D) transient inverse heat conduction problem is solved using the conjugate gradient method (CGM) and the general purpose commercial code CFX4.2-based inverse algorithm to estimate the unknown boundary heat flux in any 3-D irregular domain.

The advantage of calling CFX4.2 as a subroutine in the present inverse calculation lies in that many difficult but practical 3-D inverse problems that can be solved under this construction.

Results obtained by using the conjugate gradient method to solve these 3-D inverse problems are justified based on the numerical experiments. It is concluded that accurate boundary fluxes can be estimated by the CGM except for the final time. The reason and improvement of this singularity are addressed. Finally, the effects of the measurement errors on the inverse solutions are discussed.

1. Introduction

The direct heat conduction problems are concerned with the determination of temperature at interior points of a region when the initial and boundary conditions, thermophysical properties and heat generation are specified. In contrast, the inverse heat conduction problem involves the determination of the surface conditions [1], energy generation [2] and thermoproperties [3–5] from the knowledge of the temperature measurements taken within the body.

For the two-dimensional inverse problems in regular coordinates (i.e. the rectangular, cylindrical and spherical coordinates) or in irregular domain, various approaches are available and can be found in the literature [6–10]. However, the three-dimensional inverse problems with irregular domain is very limited in the literature.

There are many commercial codes available for solving fluid dynamic and heat transfer problems, such as CFX4.2, UNIC, PHONICS, etc. These codes can be used to calculate many practical but difficult direct thermal problems. If one can devise an inverse algorithm, which has the ability to communicate with thse commercial codes by means of data transportation, a generalized 3-D inverse heat transfer problem can thus be established. The objective of the present study is to utilize the CFX4.2 code as the subroutine in solving the 3-D inverse problems by CGM. CFX4.2 is available from AEA technology and the method of control volume is used to solve the thermal problems.

The CGM is also called an iterative regularization method, which means the regularization procedure is performed during the iterative processes and thus the determination of optimal regularization conditions is not needed. The present work addresses the developments of the conjugate gradient algorithms for estimat-

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Nomenclature

J	functional defined by Eq. (2)
J'	gradient of functional defined by Eq. (11a)
P	direction of descent defined by Eq. (3b)
q	unknown surface heat flux
S	boundary of the computational domain
T	calculated temperature
Y	measured temperature.

Greek symbols

β	search step size defined by Eq. (6)
γ	conjugate coefficient defined by Eq. (3c)
ΔT	solution of sensitivity problem
ε	convergence criteria
λ	solution for adjoint problem
σ	standard deviation of the measurement errors
ω	random number
Ω	computation domain.

Superscript

$\hat{}$	estimated values.

ing unknown boundary heat fluxes in a 3-D transient heat conduction problem. The CGM derives from the perturbation principles and transforms the inverse problem to the solution of three problems, namely, the direct, sensitivity and the adjoint problem. These three problems are solved by CFX4.2 and the calculated values are used in CGM for inverse calculations. The bridge between CFX and CGM is the INPUT/OUTPUT files. These files should be arranged such that their format can be recognized by CFX and CGM.

Finally, the inverse solutions for two transient heat conduction problems with irregular geometry and different boundary conditions will be illustrated to show the validity of using the CGM in the present 3-D inverse problem.

2. The direct problem

To illustrate the methodology for developing expressions for use in determining unknown surface heat flux in a homogeneous medium by CGM and CFX4.2, we consider the following 3-D inverse heat conduction problem. For a domain Ω, the initial temperature equals T_0. When $t > 0$, the boundary conditions on boundary surfaces S_2 (bottom surface), S_3, S_4, S_5 and S_6 are all assumed insulated, while the boundary condition on boundary S_1 (upper surface) is subjected to an unknown heat flux $q(S_1, t)$, which is a function of surface positions and time. Fig. 1a shows the geometry and the coordinates for the 3-D physical problem considered here.

The dimensionless mathematical formulation of this linear heat conduction problem is given by:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{\partial T}{\partial t} \quad \text{in}\,\Omega, \quad t > 0 \tag{1a}$$

$$\frac{\partial T}{\partial n} = 0 \quad \text{on}\, S_2\text{–}S_6, \quad t > 0 \tag{1b}$$

$$\frac{\partial T}{\partial n} = q(S_1, t) = ? \quad \text{on}\, S_1, \quad t > 0 \tag{1c}$$

$$T = T_0 \quad \text{in}\,\Omega, \quad t = 0 \tag{1d}$$

The solution for the above 3-D transient heat conduction problem in an irregular domain Ω is solved using CFX4.2 and its Fortran subroutine USRBCS. The direct problem considered here is concerned with the determination of the medium temperature when all the boundary conditions at all boundaries are known.

3. The inverse problem

For the inverse problem, the boundary heat flux on S_1 is regarded as being unknown, but everything else in Eq. (1) is known. In addition, temperature readings using infrared scanners taken at some appropriate locations and time on S_2 are considered available.

Let the temperature reading taken by infrared scanners on S_2 be denoted by $Y(S_2, t) \equiv Y(x_m, y_m, z_m, t) \equiv$

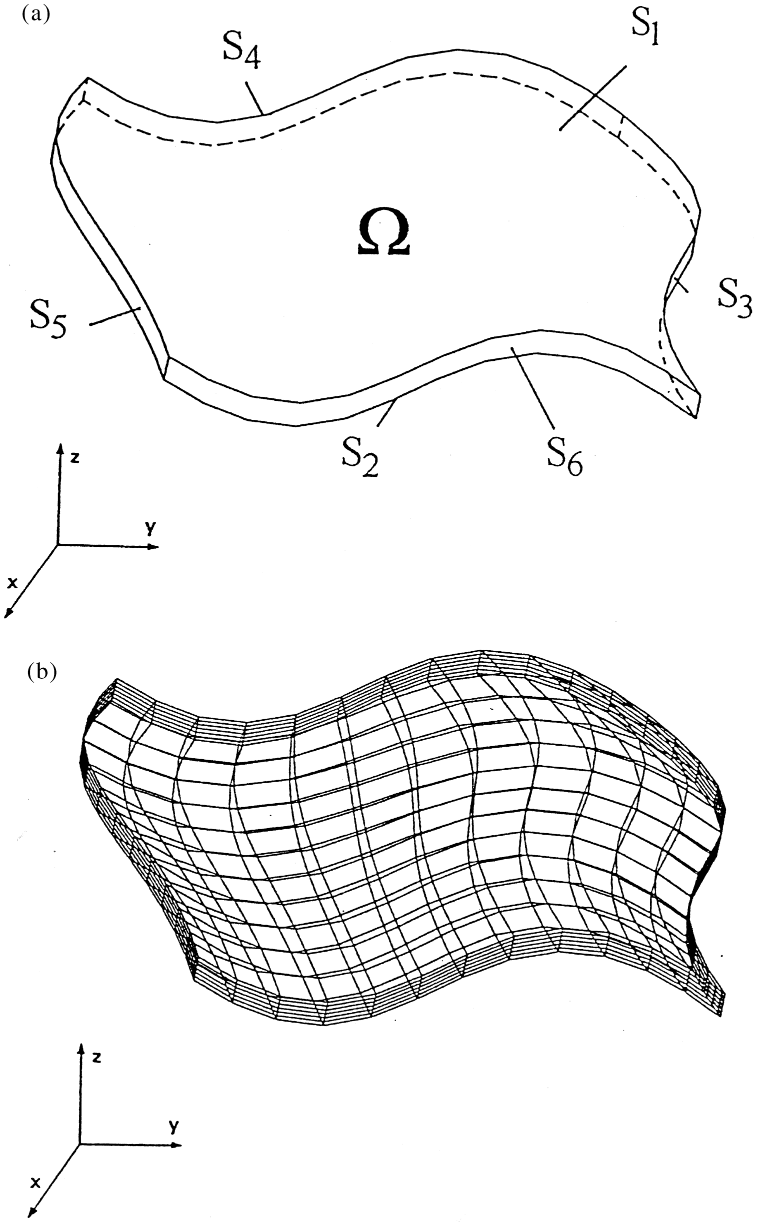


Fig. 1. (a) The geometry and coordinates for the present study. (b) The grid system for the present study.

$Y_m(t)$, $m=1$–M, where M represents the number of measured temperature extracting points. We note that the measured temperature $Y_m(t)$ contain measurement errors. Then the inverse problem can be stated as follows: by utilizing the above mentioned measured temperature data $Y_m(t)$, estimate the unknown boundary heat flux $q(S_1, t)$.

The solution of the present inverse problem is to be obtained in such a way that the following functional is minimized:

$$J[q(S_1,t)] = \int_{t=0}^{t_f} \sum_{m=1}^{M} [T(x_m, y_m, z_m, t) - Y(x_m, y_m, z_m, t)]^2 \, dt = \int_{t=0}^{t_f} \sum_{m=1}^{M} [T_m(t) - Y_m(t)]^2 \, dt \quad (2)$$

where $T_m(t)$ are the estimated or computed temperatures at the measured temperature extracting locations (x_m, y_m, z_m) at time t. These quantities are determined from the solution of the direct problem given previously by using an estimated $\hat{q}(S_1, t)$ for the exact $q(S_1, t)$. Here the hat '$\hat{}$' denotes the estimated quantities and t_f is the final time.

4. CGM for minimization

The following iterative process based on the CGM [1] is now used for the estimation of unknown heat flux $q(S_1, t)$ by minimizing the functional $J[q(S_1, t)]$

$$\hat{q}^{n+1}(S_1, t) = \hat{q}^n(S_1, t) - \beta^n P^n(S_1, t) \quad \text{for } n = 0, 1, 2, \ldots \tag{3a}$$

where β^n is the search step size in going from iteration n to iteration $n+1$, and $P^n(S_1, t)$ is the direction of descent (i.e. search direction) given by

$$P^n(S_1, t) = J'^n(S_1, t) + \gamma^n P^{n-1}(S_1, t) \tag{3b}$$

which is a conjugation of the gradient direction $J'^n(S_1, t)$ at iteration n and the direction of descent $P^{n-1}(S_1, t)$ at iteration $n-1$. The conjugate coefficient is determined from

$$\gamma^n = \frac{\int_{t=0}^{t_f}\int_{S_1}(J'^n)^2\,dS_1\,dt}{\int_{t=0}^{t_f}\int_{S_1}(J'^{n-1})^2\,dS_1\,dt} \quad \text{with } \gamma^0 = 0 \tag{3c}$$

We note that when $\gamma^n = 0$ for any n, in Eq. (3b), the direction of descent $P^n(S_1, t)$ becomes the gradient direction, i.e. the 'steepest descent' method is obtained. The convergence of the above iterative procedure in minimizing the functional J is guaranteed in [11].

To perform the iterations according to Eq. (3a), we need to compute the step size β^n and the gradient of the functional $J'^n(S_1, t)$. In order to develop expressions for the determination of these two quantities, a 'sensitivity problem' and an 'adjoint problem' are constructed as described below.

4.1. Sensitivity problem and search step size

The sensitivity problem is obtained from the original direct problem defined by Eq. (1) in the following manner: it is assumed that when $q(S_1, t)$ undergoes a variation Δq, T is perturbed by ΔT. Then replacing in the direct problem q by $q+\Delta q$ and T by $T+\Delta T$, subtracting from the resulting expressions the direct problem and neglecting the second-order terms, the following sensitivity problem for the sensitivity function ΔT are obtained.

$$\frac{\partial^2 \Delta T}{\partial x^2} + \frac{\partial^2 \Delta T}{\partial y^2} + \frac{\partial^2 \Delta T}{\partial z^2} = \frac{\partial \Delta T}{\partial t} \quad \text{in } \Omega, \quad t > 0 \tag{4a}$$

$$\frac{\partial \Delta T}{\partial n} = 0 \quad \text{on } S_2\text{–}S_6, \quad t > 0 \tag{4b}$$

$$\frac{\partial \Delta T}{\partial n} = \Delta q(S_1, t) \quad \text{on } S_1, \quad t > 0 \tag{4c}$$

$$\Delta T = 0 \quad \text{in } \Omega, \quad t = 0 \tag{4d}$$

The CFX4.2 is used to solve this sensitivity problem.

The functional $J(\hat{q}^{n+1})$ for iteration $n+1$ is obtained by rewriting Eq. (2) as

$$J(\hat{q}^{n+1}) = \int_{t=0}^{t_f} \sum_{m=1}^{M} [T_m(\hat{q}^n - \beta^n P^n) - Y_m]^2\,dt \tag{5a}$$

where we replace $\hat{q}^{n+1}$ by the expression given by Eq. (3a). If temperature $T_m(\hat{q}^n - \beta^n P^n)$ is linearized by a Taylor expansion, Eq. (5a) takes the form

$$J(\hat{q}^{n+1}) = \int_{t=0}^{t_f} \sum_{m=1}^{M} [T_m(\hat{q}^n) - \beta^n \Delta T_m(P^n) - Y_m]^2\,dt \tag{5b}$$

where $T_m(\hat{q}^n)$ is the solution of the direct problem by using estimate $\hat{q}^n$ for exact q at (x_m, y_m, z_m) and time t. The sensitivity functions $\Delta T_m(P^n)$ are taken as the solutions of problem (4) at the measured temperature extracting positions (x_m, y_m, z_m) and time t by letting $\Delta q = P^n$. The search step size β^n is determined by minimizing the functional given by Eq. (5b) with respect to β^n. The following expression results:

$$\beta^n = \frac{\int_{t=0}^{t_f} \sum_{m=1}^{M} [T_m(t) - Y_m(t)]\Delta T_m(t)\,dt}{\int_{t=0}^{t_f} \sum_{m=1}^{M} [\Delta T_m(t)^2]\,dt} \tag{6}$$

4.2. Adjoint problem and gradient equation

To obtain the adjoint problem, Eq. (1a) is multiplied by the Lagrange multiplier (or adjoint function) $\lambda(x, y, z, t)$ and the resulting expression is integrated over the correspondent space and time domains. Then the result is added to the right-hand-side of Eq. (2) to yield the following expression for the function $J[q(S_1, t)]$:

$$J[q(S_1, t)] = \int_{t=0}^{t_f} \int_{S_2} [T - Y]^2 \delta(x - x_m)\delta(y - y_m)\delta(z - z_m)\, dS_2\, dt + \int_{t=0}^{t_f} \int_{\Omega} \lambda \left\{ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} - \frac{\partial T}{\partial t} \right\} d\Omega\, dt \quad (7)$$

The variation ΔJ is obtained by perturbing q by Δq and T by ΔT in Eq. (7), subtracting from the resulting expression the original Eq. (7) and neglecting the second-order terms. We thus find

$$\Delta J = \int_{t=0}^{t_J} \int_{S_2} 2(T - Y)\Delta T \delta(x - x_m)\delta(y - y_m) \delta(z - z_m)\, dS_2\, dt + \int_{t=0}^{t_J} \int_{\Omega} \lambda \left[\frac{\partial^2 \Delta T}{\partial x^2} + \frac{\partial^2 \Delta T}{\partial y^2} + \frac{\partial^2 \Delta T}{\partial z^2} - \frac{\partial \Delta T}{\partial t} \right] d\Omega\, dt \quad (8)$$

where $\delta(\cdot)$ is the Dirac delta function and (x_m, y_m, z_m), $m = 1$–M, refer to the measured temperature extracting positions. In Eq. (8), the domain integral term is reformulated based on the Green's second identity; the boundary conditions of the sensitivity problem give by Eqs. (4b) and (4c) are utilized and then ΔJ is allowed to go to zero. The vanishing of the integrands containing ΔT leads to the following adjoint problem for the determination of $\lambda(x, y, z, t)$:

$$\frac{\partial^2 \lambda}{\partial x^2} + \frac{\partial^2 \lambda}{\partial y^2} + \frac{\partial^2 \lambda}{\partial z^2} + \frac{\partial \lambda}{\partial t} = 0, \quad \text{in}\, \Omega, \quad t > 0 \quad (9a)$$

$$\frac{\partial \lambda}{\partial n} = 0 \quad \text{on}\, S_1, S_3\text{–}S_6, \quad t > 0 \quad (9b)$$

$$\frac{\partial \lambda}{\partial n} = 2(T - Y)\delta(x - x_m)\delta(y - y_m)\delta(z - z_m) \quad \text{on}\ S_2, \quad t > 0 \quad (9c)$$

$$\lambda = 0 \quad \text{in}\, \Omega, \quad t = t_f \quad (9d)$$

The adjoint problem is different from the standard initial value problems in that the final time conditions at time $t = t_f$ is specified instead of the customary initial condition. However, this problem can be transformed to an initial value problem by the transformation of the time variables as $\tau = t_f - t$. Then the standard techniques of BEM can be used to solve the above adjoint problem.

Finally, the following integral term is left

$$\Delta J = \int_{t=0}^{t_f} \int_{S_1} \lambda \Delta q(S_1, t)\, dS_1\, dt \quad (10a)$$

From definition [1], the functional increment can be presented as

$$\Delta J = \int_{t=0}^{t_f} \int_{S_1} J'[q(S_1, t)]\Delta q(S_1, t)\, dS_1\, dt \quad (10b)$$

A comparison of Eqs. (10a) and (10b) leads to the following expression for the gradient of functional $J'[q(S_1, t)]$ of the functional $J[q(S_1, t)]$:

$$J'[q(S_1, t)] = \lambda(x, y, z)\, |_{\text{on}\, S_1} \quad (11a)$$

We note that the gradient J' at final time $t = t_f$ is always equal to zero since $\lambda(x, y, z, t_f) = 0$. If the initial guess values of q^0 cannot be predicted correctly before the inverse calculation, the estimated values of heat flux q will deviate from exact values near the final time conditions. This is the case in the present study! Now the artificial gradient at final time is defined as follows:

$$J'(S_1, t_f) = \lambda(x, y, z, t_f - \Delta t) \quad (11b)$$

where Δt denotes the time increment used in CFX.

By replacing the artificial gradient Eq. (11b) to the gradient Eq. (11a), the singularity at final time $t = t_f$ can be avoided in the present study and a reliable inverse solution can be obtained.

4.3. Stopping criterion

If the problem contains no measurement errors, the traditional check condition is specified as

$$J\lfloor \hat{q}^{n+1}(S_1, t) \rfloor < \varepsilon \quad (12a)$$

where ε is a small-specified number. However, the observed temperature data may contain measurement errors. Therefore, we do not expect the functional Eq. (2) to be equal to zero at the final iteration step. Following the experiences of the authors [1–3], we use the discrepancy principle as the stopping criterion, i.e. we assume that the temperature residuals may be approximated by

$$T_m(t) - Y_m(t) \approx \sigma \quad (12b)$$

where σ is the stand deviation of the measurements, which is assumed to be a constant. Substituting Eq. (12b) into Eq. (2), the following expression is obtained for stopping criteria ε:

$$\varepsilon = M\sigma^2 t_f \quad (12c)$$

Then, the stopping criterion is given by Eq. (12a) with ε determined from Eq. (12c).

5. Computational procedure

The computational procedure for the solution of this inverse problem using CGM may be summarized as follows.

Suppose $\hat{q}^n(S_1, t)$ is available at iteration n.
Step 1. Solve the direct problem given by Eq. (1) for $T(x, y, z, t)$.
Step 2. Examine the stopping criterion given by Eq. (12a) with ε given by Eq. (12c). Continue if not satisfied.
Step 3. Solve the adjoint problem given by Eq. (9) for $\lambda(x, y, z, t)$.
Step 4. Compute the gradient of the functional J' from Eq. (11).
Step 5. Compute the conjugate coefficient γ^n and direction of descent P^n from Eqs. (3c) and (3b), respectively.
Step 6. Set $\Delta q = P^n$, and solve the sensitivity problem given by Eq. (4) for $\Delta T(x, y, z, t)$.
Step 7. Compute the search step size β^n from Eq. (6).
Step 8. Compute the new estimation for $\hat{q}^{n+1}$ from Eq. (3a) and return to Step 1.

6. Results and discussion

The objective of this article is to show the validity of the CGM in estimating the boundary heat flux $q(S_1, t)$ accurately with no prior information on the functional form of the unknown quantities.

To illustrate the accuracy of the CGM in predicting boundary heat flux $q(S_1, t)$ in an arbitrary domain Ω with 3-D inverse analysis from the knowledge of transient temperature recordings, two specific examples having different forms of heat fluxes are considered here.

In order to compare the results for situations involving random measurement errors, we assume normally distributed uncorrelated errors with zero mean and constant standard deviation. The simulated inexact measurement data $\mathbf{Y}$ can be expressed as

$$\mathbf{Y} = \mathbf{Y}_{\text{exact}} + \omega\sigma \tag{13}$$

where $\mathbf{Y}_{\text{exact}}$ is the solution of the direct problem with an exact boundary heat flux $q(S_1, t)$; σ is the standard deviation of the measurements; and ω is a random variable generated by subroutine DRNNOR of the IMSL [12] and will be within −2.576–2.576 for a 99% confidence bound.

One of the advantages of using the conjugate gradient method to solve the inverse problems is that the initial guesses of the unknown quantities can be chosen arbitrarily. In all the test cases considered here the initial guesses of $\hat{q}(S_1, t)$ is taken as $\hat{q}(S_1, t)_{\text{initial}} = 0.0$.

The geometry for the test case is shown in Fig. 1a, which represents an arbitrarily irregular domain having thin thickness in the z-direction. If this thickness is too thick, the estimated fluxes may be damped. The boundary conditions on S_2 (bottom surface), S_3, S_4, S_5 and S_6 are all insulated while an unknown heat flux $q(S_1, t)$ is prescribed on S_1 (upper surface). The grids along the x- and y-directions are taken as 12 while along the z-direction they are taken as 6. The time interval is chosen as 24, i.e. $t_f = 24$, and time step $\Delta t = 1$ is used. Therefore, a total of 3456 unknown discreted heat fluxes are to be determined in the present study. The number of measured temperature extracting positions M is taken as 144. The grid system for the present study is shown in Fig. 1b.

We now present below the numerical experiments in determining $q(S_1, t)$ by the inverse analysis using the CGM.

(A) Numerical Test Case 1

The unknown transient boundary heat flux $q(S_1, t)$ on S_1 is assumed as

$$q_1(I, J, t) = 60 \times \sin\left(\frac{t}{t_f}\pi\right), \quad \begin{matrix} 1 \le I \le 12 \\ 1 \le J \le 12 \end{matrix} \quad 24 \ge t \ge 0$$

$$q_2(I, J, t) = 60 \times \sin\left(\frac{t}{t_f}\pi\right), \quad \begin{matrix} 3 \le I \le 10 \\ 3 \le J \le 10 \end{matrix} \quad 24 \ge t \ge 0$$

$$q_3(I, J, t) = 40 \times \sin\left(\frac{t}{t_f}\pi\right), \quad \begin{matrix} 5 \le I \le 8 \\ 5 \le J \le 8 \end{matrix} \quad 24 \ge t \ge 0$$

$$q(I, J, t) = (q_1 + q_2 + q_3), \quad \text{in } \Omega \quad 24 \ge t \ge 0 \tag{14}$$

where I and J represent the grid index on surface S_1. It is obvious from Eq. (14) that $q(S_1, t_f) = 0$ due to sinusoidal function. Since $\hat{q}(S_1, t)_{\text{initial}} = 0.0$, we concluded that the singularity at final time t_f will not happen in this case and accurate inverse solutions can be obtained. The exact plot for $q(S_1, t)$ at $t = 6$ and 12 is shown in Fig. 2.

The inverse analysis is first performed by assuming exact measurements, $\sigma = 0.0$. The estimated $q(S_1, t)$ after 30 iterations at $t = 6$ and 12 is shown in Fig. 3. It can be seen from Figs. 2 and 3 that the estimations are accurate except for the locations of discontinuity. The average error for this case is calculated as 5.2% where

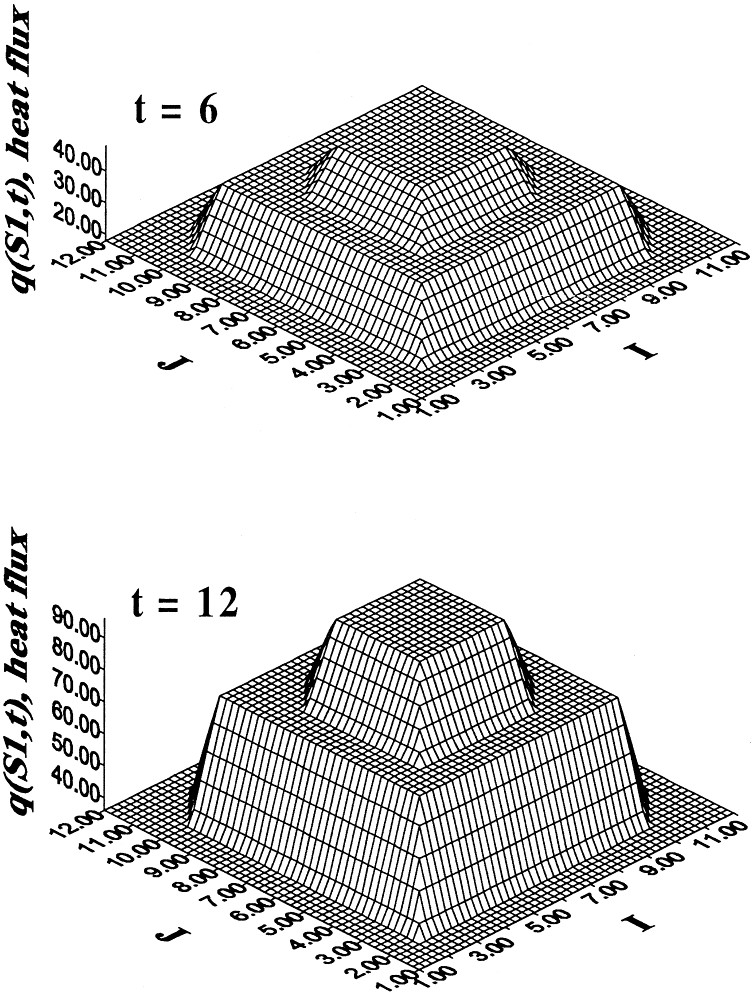


Fig. 2. The exact heat fluxes $q(S_1, t)$ for case 1 at $t=6$ and 12.

the average error for the estimated heat flux is defined as

$$\text{Average error }\% = \left[\sum_{I=1}^{12}\sum_{J=1}^{12}\sum_{t=1}^{t_f}\left|\frac{q(I,J,t)-\hat{q}(I,J,t)}{q(I,J,t)}\right|\right] \div (12\times 12\times t_f)\times 100\% \tag{15}$$

where I and J represent the index of discreted unknown heat fluxes on S_1 and t denotes the index of discreted time, while $q(I, J, t)$ and $\hat{q}(I, J, t)$ denote the exact and estimated values of boundary heat flux.

Next, let us discuss the influence of the measurement errors on the inverse solutions. First, the measurement error for the temperatures measured by infrared scanners is taken as $\sigma=0.35$ (about 1% of the average measured temperature), then error is increased to $\sigma=0.7$ (about 2% of the average measured temperature). The estimated $q(S_1, t)$ at $t=6$ and 12 is shown in Figs. 4 and 5 where the average errors in Fig. 4 is 7.1% and in Fig. 5 is 8.3%. The stopping criteria ε is calculated from Eq. (12c) and the number of iterations is about 15 for the above test cases. This implies that reliable inverse solutions can still be obtained when measurement errors are considered.

In order to show the estimated inverse solutions more clearly, we plot Fig. 6 which is the estimated $q(S_1, t)$ at $t=6$ and 12 obtained from Figs. 3 and 5 at $J=6$.

(B) Numerical Test Case 2

The geometry for this test case is the same as was used in Test Case 1. The unknown boundary heat flux $q(S_1, t)$ is assumed as

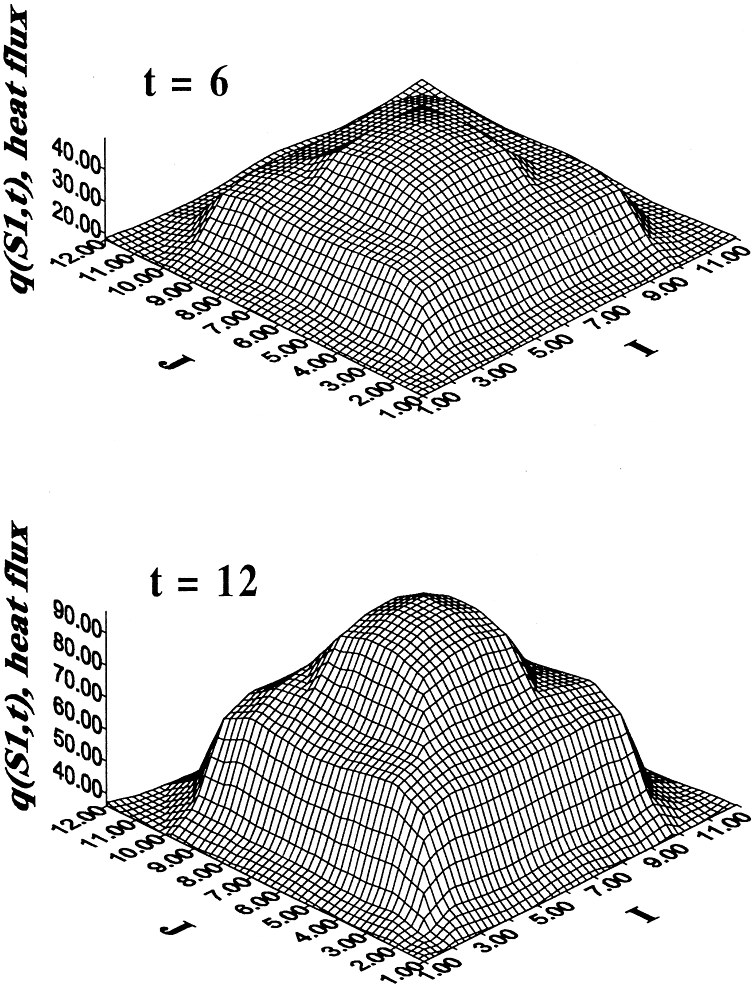


Fig. 3. The estimated heat fluxes $q(S_1, t)$ at $t=6$ and 12 using $\sigma=0.0$.

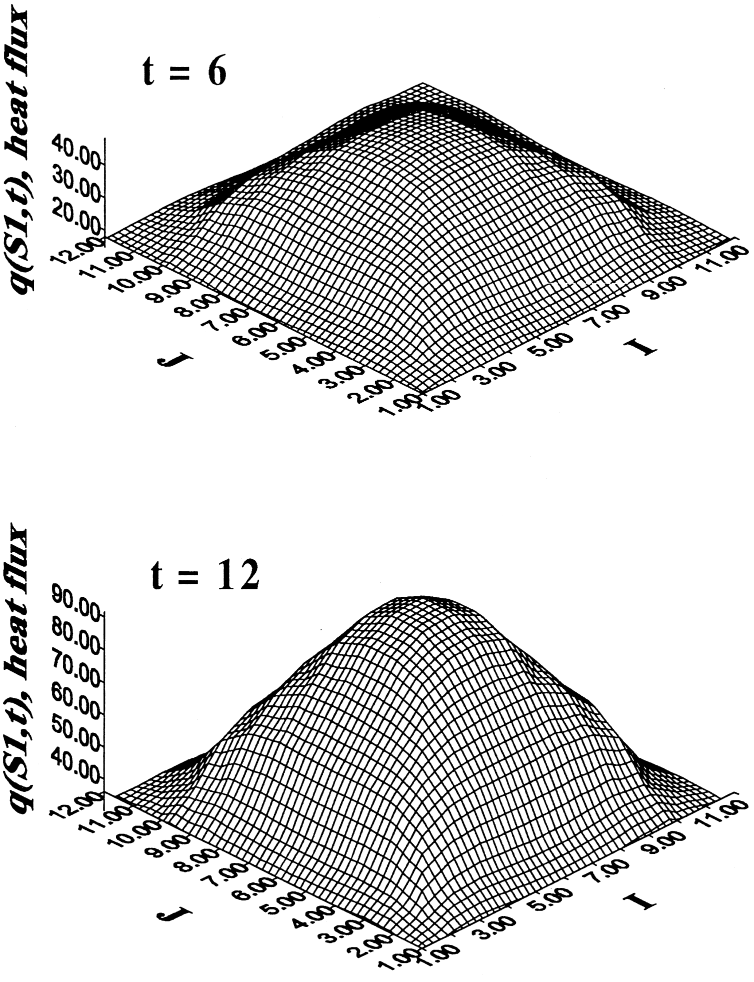


Fig. 4. The estimated heat fluxes $q(S_1, t)$ at $t = 6$ and 12 using $\sigma = 0.35$.

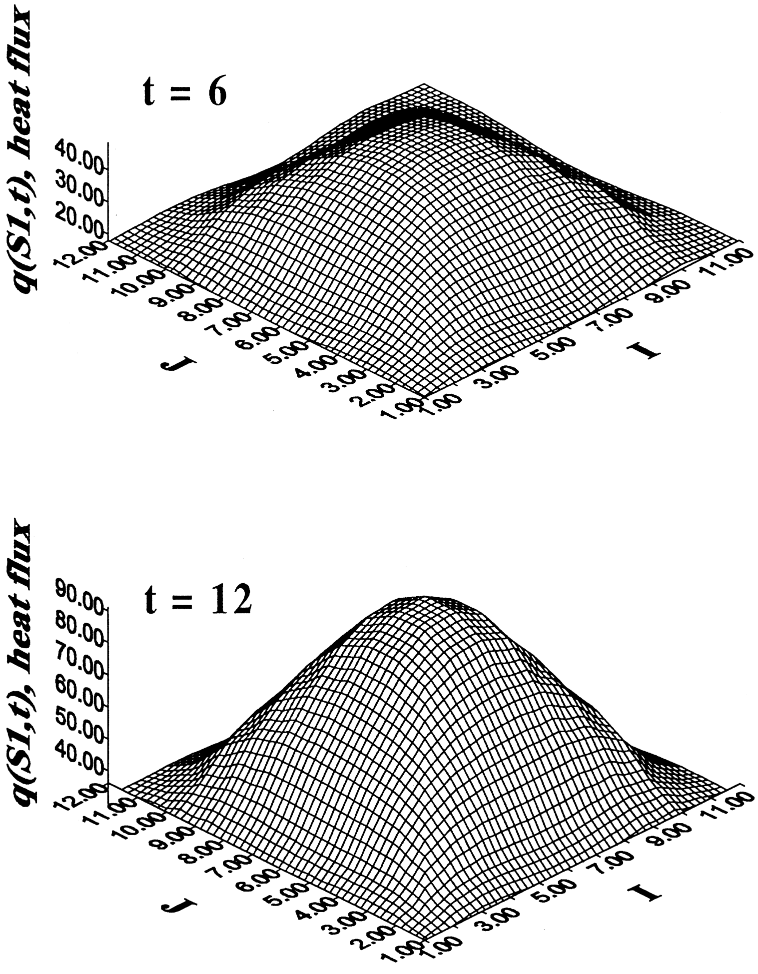


Fig. 5. The estimate heat fluxes $q(S_1, t)$ at $t=6$ and 12 using $\sigma=0.7$.

$q_1(I) = [-0.08(I-4)^2 + 2] \quad 1 \le I \le 12;$ If $q_1 < 0$ then $q_1 = 0$

$q_2(J) = [-0.08(J-4)^2 + 2] \quad 1 \le J \le 12;$ If $q_2 < 0$ then $q_2 = 0$

$q_3(I) = [-0.16(I-9)^2 + 1.6] \quad 1 \le I \le 12;$ If $q_3 < 0$ then $q_3 = 0$

$q_4(J) = [-0.16(J-10)^2 + 1.6] \quad 1 \le J \le 12;$ If $q_4 < 0$ then $q_4 = 0$

$$q_5(I, J, t) = [q_1 \times q_2] \times \left(\frac{t}{2.2}\right) \quad \begin{matrix} 1 \le I \le 12 \\ 1 \le J \le 12 \end{matrix} \quad 24 > t > 0$$

$$q_6(I, J, t) = [q_3 \times q_4] \times \left(\frac{t}{1.2}\right) \quad \begin{matrix} 1 \le I \le 12 \\ 1 \le J \le 12 \end{matrix} \quad 24 > t > 0$$

$$q(I, J, t) = q_1(I, J, t) + q_2(I, J, t) + 50 \quad \begin{matrix} 1 \le I \le 12 \\ 1 \le J \le 12 \end{matrix} \quad 24 > t > 0 \tag{16}$$

The exact plot for $q(S_1, t)$ at $t=6$ and 12 is shown in Fig. 7. One should note that in Test Case 2 we still use $\hat{q}(S_1, t)_{\text{initial}} = 0.0$, but now $q(S_1, t_f) \neq 0$, therefore, we concluded that the singularity at final time t_f will happen in this case and the modified gradient at final time in Eq. (11b) must be used to overcome this singularity. However, the inverse solutions near final time under this consideration is still not accurate, therefore, the estimated heat flux at the last few time steps are going to be discarded to ensure good estimations are obtained.

In Test Case 2 the estimated $\hat{q}(S_1, t)$ is chosen up to $t=20$ and the remainder four-time steps are neglected. The inverse problem with CGM is first calculated by using exact measurements, i.e. $\sigma=0.0$. After 30 iterations the estimated boundary heat flux $q(S_1, t)$ at $t=6$ and 12 is shown in Fig. 8. It is obvious that the accuracy of the inverse solutions is reliable since the average errors for CGM is 2.0% in this case.

Next, consider measurement errors $\sigma=1.4$ (about 2.0% of the average measured temperature) and measurement errors $\sigma=2.0$ (about 3.0% of the average measured temperature), the inverse solutions are shown in Figs. 9 and 10. The average errors for CGM are 2.8 and 3.7% in Figs. 9 and 10, respectively.

In order to show the estimated inverse solutions more clearly, Figs. 11 and 12 show the estimated $q(S_1, t)$ at $t=6$ and 12 obtained from Figs. 8 and 10 at $J=4$ and 9, respectively.

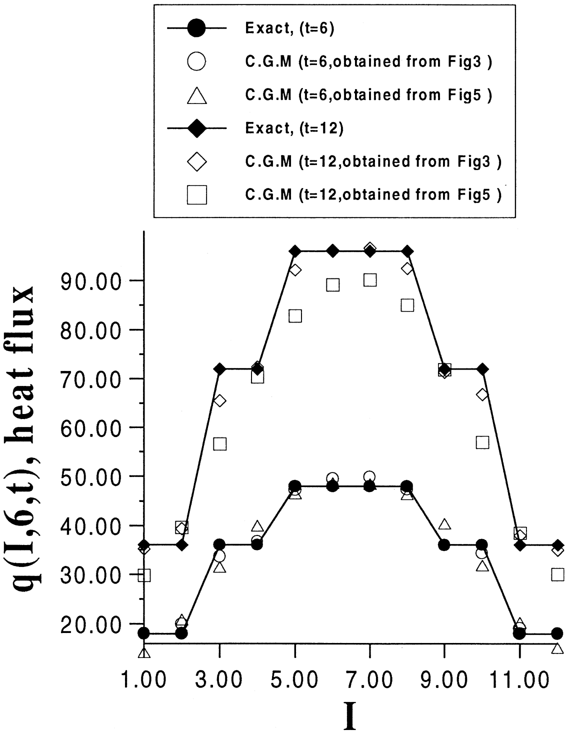


Fig. 6. The estimated heat fluxes $q(S_1, t)$ extracting from Figs. 3 and 5 at $t=6$, 12 and $J=6$.

From the above two test cases we learned that a 3-D inverse heat conduction problem in estimating boundary heat flux is now completed. Reliable estimations can be obtained when using either exact or error measurements.

7. Conclusions

The CGM along with the CFX4.2 was successfully applied for the solution of the 3-D inverse heat conduction problem to determine the unknown transient boundary heat flux in an irregular domain by utilizing simulated temperature readings obtained from infrared scanners. Several test cases involving different measurement errors and heat fluxes were considered. The results show that the inverse solutions obtained by CGM remain stable and regular as the measurement errors are increased.

From the numerical test cases in the present study

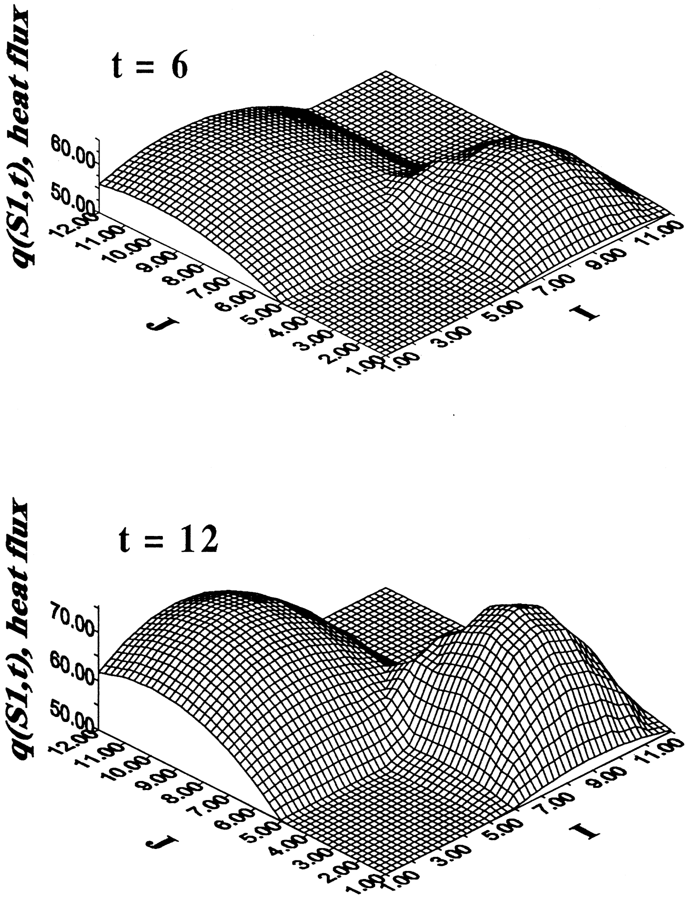
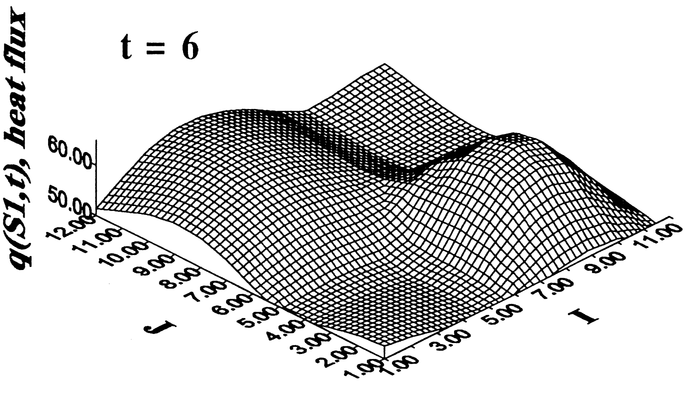


Fig. 7. The exact heat fluxes $q(S_1, t)$ for Case 2 at $t=6$ and 12.



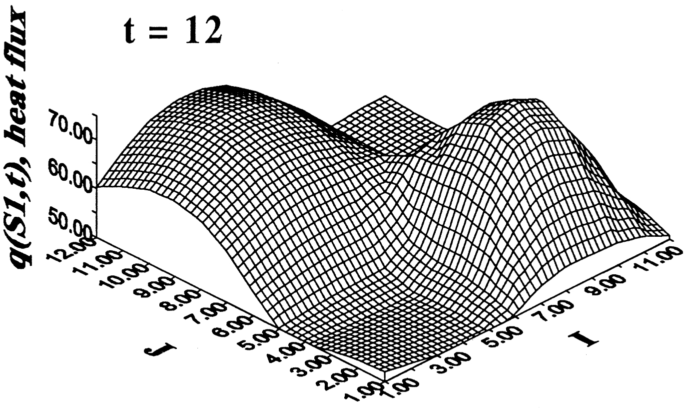


Fig. 8. The estimated heat fluxes $q(S_1, t)$ at $t=6$ and 12 using $\sigma=0.0$.

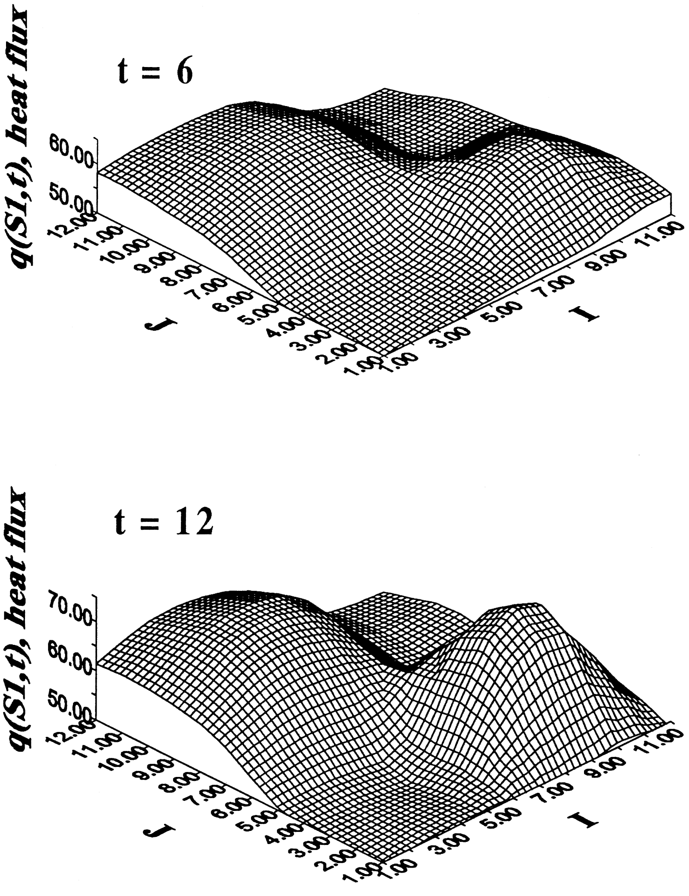


Fig. 9. The estimated heat fluxes $q(S_1, t)$ at $t = 6$ and 12 using $\sigma = 1.4$.

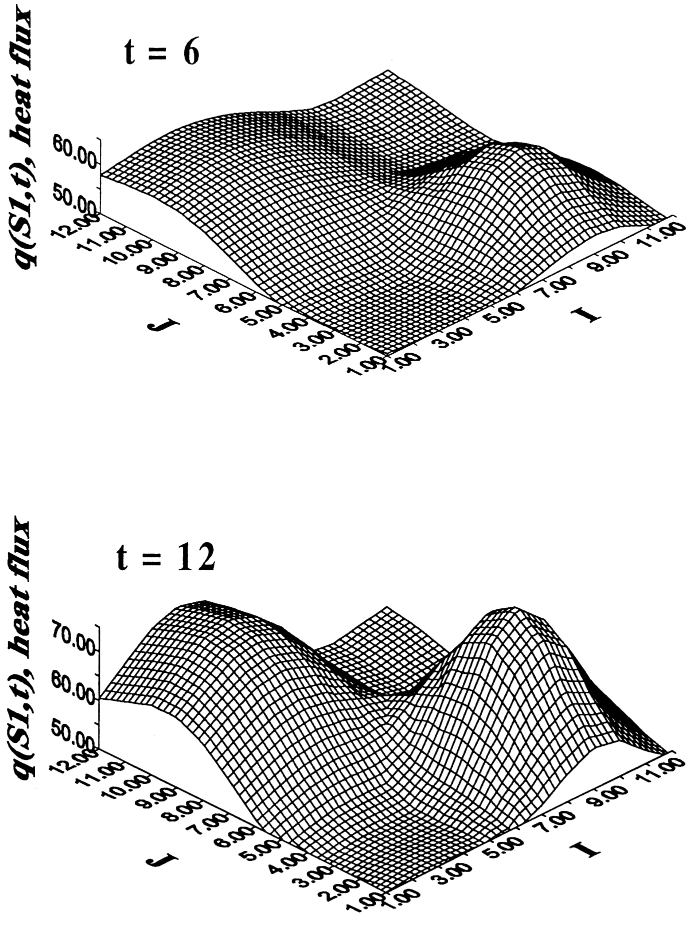


Fig. 10. The estimated heat fluxes $q(S_1, t)$ at $t = 6$ and 12 using $\sigma = 2.0$.

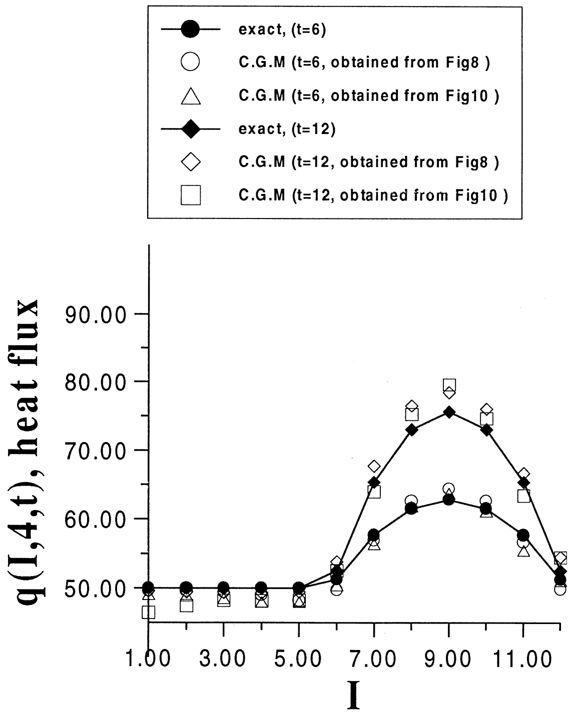


Fig. 11. The estimated heat fluxes $q(S_1, t)$ extracting from Figs. 8 and 10 at $t=6$, 12 and $J=4$.

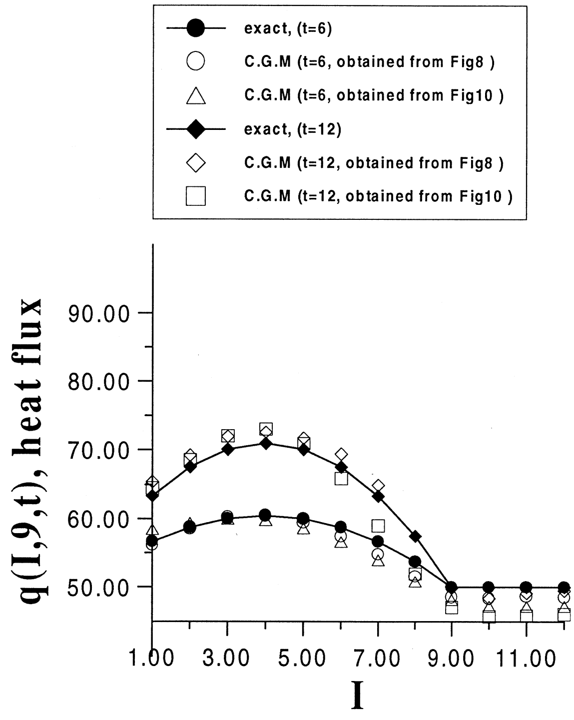


Fig. 12. The estimated heat fluxes $q(S_1, t)$ extracting from Figs. 8 and 10 at $t=6$, 12 and $J=9$.

we concluded that the use of CFX as the subroutine in the 3-D inverse problem in estimating the unknown boundary heat flux with the CGM has been done successfully. By using the same algorithm, many practical but difficult 3-D inverse problems can also be solved.

Acknowledgements

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